

Quiz 14

October 26, 2016

Show all work and circle your final answer.

1. If $y = e^{\sin \pi x}$, use differentials to estimate the change in y as x increases from 2 to 3. $\rightarrow x=2, \Delta x = dx = 1$

$$dy = e^{\sin(\pi x)} \cdot \cos(\pi x) \cdot \pi \, dx$$

$$\Delta y \approx dy = e^{\sin(2\pi)} \cos(2\pi) \pi (1)$$

$$= e^0 (1) (\pi) = \boxed{\pi}$$

Note:

we can also compute the actual change:

$$\Delta y = e^{\sin(3\pi)} - e^{\sin(2\pi)} = 0.$$

So this isn't a great estimate!

2. Estimate $2\sqrt{100.1}$ using a linearization (write your answer as a decimal).

Find the linearization of $f(x) = 2\sqrt{x}$ at $a=100$:

$$f'(x) = \frac{1}{\sqrt{x}} \text{ so } f'(100) = \frac{1}{10}$$

$$f(100) = 2\sqrt{100} = 20$$

$$\text{So } L(x) = 20 + \frac{1}{10}(x-100)$$

$$L(100.1) = 20 + \frac{1}{10}(100.1-100) = 20 + \frac{1}{100} = \boxed{20.01}$$

3. Find the maximum value of $f(x) = x^3 - 3x^2 - 9x + 27$ on the interval $[-2, 0]$. Critical points:

$$f'(x) = 3x^2 - 6x - 9 \stackrel{\text{set}}{=} 0$$

Points:

$$3(x^2 - 2x - 3) = 0$$

$$3(x-3)(x+1) = 0$$

$$x = -1 \text{ OR } 3 \leftarrow \text{not in } [-2, 0].$$

x	$f(x)$
-2	$-8 - 12 + 18 + 27 = 25$
-1	$-1 - 3 + 9 + 27 = 32 \leftarrow \text{max}$
0	$0 + 0 + 0 + 27 = 27$